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Exchange-Rate Volatility, Exchange-Rate Disconnect, and the Failure of Volatility Conservation

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Abstract

Empirical analysis of exchange rates has produced puzzles that conventional models of exchange rates cannot explain. Here we deal with four puzzles regarding both real and nominal exchange rates, which are robust and inconsistent with standard theory. These puzzles are that both real and nominal exchange rates: i) are disconnected from fundamentals, ii) are much more volatile than fundamentals, iii) show little difference in behavior, and iv) fail to satisfy conservation of volatility. We develop a two-country, two-currency version of the random matching model to study exchange rates. We show that search and legal restrictions can produce exchange-rate dynamics consistent with these four puzzles.

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1 Introduction

Thus far economists have succeeded in putting up the list of exchange-rate puzzles. Both nominal and real exchange rates i) are pretty much unrelated to fundamentals (determination puzzle), ii) are much more volatile than fundamentals (excess volatility puzzle), iii) show little difference in behavior,¹ and iv) fail to satisfy volatility conservation.² The latter means that contrary to predictions of standard models, an exchange rate regime switch has no consequences to the volatility of fundamentals such as output or money supply. Here we present a model of exchange-rate uncertainty, which is consistent with these four puzzles.³

The existing literature on exchange-rate volatility seems to evolve along two dimensions. On one side there are models, where exchange-rate uncertainty is non-fundamental; in such models exchange-rate dynamics is driven by an extraneous sunspot variable.⁴ These models are motivated by a belief that at least over short horizons most of exchange-rate volatility comes from sources other than fundamentals; generally, that literature makes an extensive use of substitutability among currencies. Extreme substitutability allows to produce nominal exchange-rate uncertainty even when fundamentals remain fixed, yet it provides little help in explaining the behavior of real exchange rates.⁵

On the other side there are models, where exchange-rate volatility is a reflection of the volatility of fundamentals (most frequently of the money supply). These models introduce frictions into otherwise perfect markets; the role of the frictions is to transform the relatively small volatility of ob-

¹See, e.g. Obstfeld and Rogoff (1996).

²Empirical failure of volatility conservation is documented, for example, in Flood and Rose (1995).

³Some of the puzzles we mention on our list have been recognized in other markets, e.g. in equity market. As regards equities, determination and excess volatility puzzles were recognized by Roll (1988) and Shiller (1981) respectively.

⁴The list of such models includes Kareken and Wallace (1981), Manuelli and Peck (1990), King, Wallace, and Weber (1992), Barnett (1992), Alonso (2004).

⁵There is only one paper in that literature, Barnett (1992), which attempts to deal with real exchange rates. Barnett extends King, Wallace, and Weber (1992) by adding legal restrictions, which allows to produce many dynamic equilibria consistent with disconnected and volatile real exchange rates. However, in all of such equilibria exchange-rate movements vanish over time, so that the sunspot becomes irrelevant in the long run. That point has been extensively emphasized by Alonso (2004).

servable macroeconomic variables to exchange rates, so that the resulting exchange-rate volatility becomes large enough to be consistent with the data. Such frictions include search, asymmetric information about fundamentals, and noisy (erroneous) expectations.⁶ These models produce exchange-rate dynamics consistent with the determination and excess volatility puzzles, however they generally fail to address the other two exchange rate puzzles.

There are two papers in that literature which are consistent with failure of volatility conservation: Engel and Devereux (2002) and Jeanne and Rose (2002). Engel and Devereux assume local currency pricing and heterogeneous product distribution, which helps to reduce the pass-through from exchange rates to prices. They also make use of noise traders in international asset markets, which breaks down arbitrage opportunities in these markets.⁷ Under some conditions that combination makes exchange rates be highly volatile and disconnected from the real economy, so that large exchange-rate movements matter little for real variables.

Here we show that search and legal restrictions can produce exchange-rate dynamics consistent with these four puzzles. We introduce small shocks to prices in goods markets, which generate large fluctuations in exchange rates under floating rates. Such shocks are a standard ingredient of models of non-fundamental exchange-rate uncertainty, but they can also be regarded as some kind of markup shocks. Under fixed rates the model is consistent with real exchange rates, which fluctuate only to the extent of small shocks to prices in goods markets. This happens because search frictions eliminate the pass-through from exchange rates to prices; the role of search is to create a range of international relative prices consistent with willingness to trade by risk averse individuals. Within that range exchange-rate fluctuations do not pass to goods prices and hence do not matter the real economy. In that regard Engel and Devereux (2002) is a close precursor to what we do.

We start from existing work on matching models with multiple currencies.

⁶See Alessandria (2003, 2004), Lyons (2001), Jeanne and Rose (2002), and others.

⁷That assumption is extensively used in Jeanne and Rose (2002). In addition to noisy expectations Jeanne and Rose assume entry costs to forex market. In their model noise traders refrain from entering the market in case of fixed rates because they cannot make enough profit from speculation in a low volatility environment. Then, given that a larger fraction of traders make correct expectations of returns from speculation, the volatility is low. Under floating rates noise traders expect high volatility and enter the market. Given that a lot of traders make noisy expectations, the volatility is high. Thus, the model has multiple equilibria, which display different amounts of volatility vis-a-vis constant volatility of fundamentals.

Our model is most closely related to that in two papers by Trejos and Wright (1996, 2001). The model, however, differs from theirs in three main respects. First, we have a country-specific cash-in-advance constraint in goods trade.⁸ Second, we have an aggregate country-specific sunspot (mark-up shock) in goods trade that affects the way the gains from trade are divided between the seller of the good and the buyer. Third, there is a separate round of pairwise trade which is restricted to trade of one currency for another; we assume that realization of the sunspot is known prior to that round of trade.

In particular, the sequence of actions in a period is the following. Each person starts a period with home money. Then, individuals receive idiosyncratic preferences shocks which determine whether or not they want foreign goods that period. After that, people learn how the gains from trade in goods markets will be divided in that period. We assume that there are separate shocks in home and foreign markets, so that home and foreign prices are stochastic and are drawn from a known joint distribution.⁹ Then, those who want foreign goods enter a foreign exchange market. Under fixed rates, they can simply trade one currency for the other at a constant (e.g., one-to-one) rate. Under floating rates this market has pairwise meetings. Each pair in a meeting plays an alternating-offer bargaining game; once an agreement about the exchange rate is reached, the pair proceeds to goods market. We thus assume different ways to divide the gains from trade in goods markets and in foreign exchange market. In doing so we are motivated by observation that relative prices account for most of exchange-rate volatility.¹⁰ Therefore, it seems important to attenuate the pass-through from exchange rates to goods prices, yet to keep a close link from goods prices to exchange rates. After trade in foreign exchange people meet pairwise to trade money for goods, subject to the constraint that foreign goods must be purchased with foreign money. At the end of the period, each person has only home money.

Because goods prices comprise the value of money, uncertainty over division of the gains from trade causes fluctuations in the value of money. These

⁸That alone would seem to limit our ability to account for nominal exchange rate uncertainty because it limits currency substitution.

⁹We do not stick to a particular model of trade — monopolistic competition or other — our only requirement is that trades in goods markets satisfy *ex post* individual rationality in meetings. Ravikumar and Wallace (2001) and Wallace and Zhu (2003) require all trades be in pairwise core. Here, for the sake of tractability, we do not require efficiency; however all of our results apply if the core is imposed.

¹⁰See, e.g. Engel (1999).

fluctuations affect the bargaining position of individuals in foreign exchange meetings and, therefore, pass onto exchange rates. Provided that individuals are patient and that their marginal valuation of consumption is large enough, resulting nominal exchange-rate volatility is much larger than price volatility. At the same time, because search frictions attenuate the pass-through from exchange rates to prices, real variables fluctuate only to the extent of small shocks to prices. This makes exchange rates seem disconnected from the real economy. Moreover, real exchange rates are almost as volatile as nominal exchange rates, so that there is little difference in behavior between the two. Fixing floating rates implies that real exchange volatility diminishes to the order of price volatility. Because there is no pass-through, that is accompanied by no change in volatility of real variables, which is consistent with empirics documented in Baxter and Stockman (1989) and Flood and Rose (1995).

Because we want to obtain closed-form solutions, we present a version with indivisible currencies. As is by now standard in models with indivisible money, we introduce lotteries over the transfers of money as a way to approximate divisibility.¹¹ However, we are confident that all our findings extend into environments with perfectly divisible money. Roughly, indivisibility makes the value function be linear and discretizes the support of the distribution of money. Because the uncertainty over division of the gains from trade shifts the entire value function, its shape (linear vs. strictly concave in case of divisible money) is not important.

The rest of the paper is organized as follows. In the next section we describe the environment; section 3 provides a discussion of fixed exchange-rate regime; section 4 offers a discussion of floating exchange-rate regime; section 5 concludes. All proofs and numerical examples are in appendix.

2 Environment

There are two countries A and B populated by a $[0, 1]$ -continuum of infinitely lived agents. Time is discrete and the horizon is infinite. In every period agents can produce and consume a subset of $2N$ non-storable perfectly divisible goods. An agent of type n consumes either domestic good n or each of N types of foreign goods, and produce only domestic good $n + 1$

¹¹See Berentsen, Molico, and Wright (2001) for the treatment of lotteries and further discussion.

(modulo N). We assume that $N \geq 3$, in which case all domestic meetings are at most single-coincidence-of-wants meetings. Each person maximizes expected discounted utility with discount factor $\beta \in (0, 1)$. Utility in a period is given by $u(x) - y$, where x denotes consumption and y denotes production of an individual. We assume that $u'(x) > 0$, $u''(x) < 0$ for all $x > 0$, that $u(0) = 0$, and that there exists $\hat{x} > 0$ such that $u(\hat{x}) = \hat{x}$.

There are two indivisible fiat currencies A and B . Money A is a legal tender in country A and money B is a legal tender in country B . Initially, a fraction $m_i \in (0, 1)$ of the population in the country i is endowed with one unit of money i . Both monies cannot be consumed or produced by any private individual (they can only be used as media of exchange). Agents with money can buy consumption goods (they will be referred to as buyers), while agents without money can only produce goods (they will be referred to as sellers).

In every period each individual meets another person at random to trade goods for money. When buyers and sellers meet in trade meetings,¹² they trade at prices, which may vary across meetings.¹³ Because consumption goods are perfectly divisible, it is convenient to normalize prices such that the buyer trades the entire stock of her money (that is, given indivisibility of money, money changes hands with probability one).

We assume that all trades in goods markets satisfy ex post individual rationality. In every single-coincidence meeting a pair faces a randomly drawn price to which both buyer and seller say either yes or no. If both say yes, then the seller produces consumption good in amount worth a unit of buyer's money, both good and money change hands, and the two individuals part; if at least one says no, then nothing happens in a meeting which then dissolves. Ex post individual rationality implies that the two individuals agree to trade if, given price draw, the gains from trade are nonnegative.

To simplify the matters, we assume substantial coordination of prices across meetings. Price coordination is achieved via realizations of an aggregate sunspot variable, which can also be regarded as some kind of a markup shock. Specifically, we assume that there is a separate shock for home country and for foreign country, so that at any given date sellers from country i face a uniform price p_i . As regards time-series price changes, we assume that pairs

¹²A trade meeting is a single-coincidence meeting in which the buyer likes the good produced by the seller.

¹³One meaning of such a randomness is to emphasize that individuals experience various production and consumption opportunities.

of prices $\mathbf{p} \equiv (p_A, p_B)$ are independently drawn from a given distribution μ , which is common knowledge among all individuals.

In every period, a measure $\gamma \in [0, \bar{\gamma}]$, $\bar{\gamma} < \min(m_A, m_B)$, of citizens of each country meet foreigners. There are several interpretations of international trade arrangements consistent with our results. One of such interpretations is tourism: some individuals receive a period preference shock, so that they are willing to consume foreign goods. Without any changes to substance, we assume that these are domestic buyers. Buyers travel abroad, however because domestic money cannot be used as payment in a foreign country, they must obtain foreign money before making their purchase.

Tourists can exchange money in a special place on the border, which we refer to as currency exchange. The currency exchange has pairwise meetings; given realization \mathbf{p} of current prices, the post-trade allocation of money in every currency exchange meeting is determined as an outcome of a bilateral trade mechanism. Because money is indivisible, post-trade money holdings of any individual are in discrete set $\mathbb{S} \equiv \{0, A, B, AB\}$. Thus, any sensible mechanism should have randomization over the transfers of money, or lotteries.¹⁴ The outcome of a lottery is a prescription of who of the two individuals in a meeting gets what combination of money from \mathbb{S} subject to the constraint that no money is either created, destroyed, or hoarded. Without changes to substance, we limit ourselves to lotteries, which are consistent with independent transfers of money in currency exchange meetings. All such lotteries are described by pairs $\boldsymbol{\tau}$ of real numbers, $\boldsymbol{\tau} \equiv (\tau_A, \tau_B)$, where $\tau_i \in [0, 1]$ is the probability that money i changes hands.

Here we consider two different mechanisms, which correspond to “fixed” and “floating” exchange-rate regimes. Under fixed exchange-rate regime a lottery $\boldsymbol{\tau}$ is set by the government and is a must to all individuals in the currency exchange (in the sense that every pair either draws an outcome from that lottery or refuses to trade). Under floating exchange-rate regime each pair bargains about lotteries prior to allocating their money in accordance with an outcome drawn from the agreed upon lottery $\boldsymbol{\tau}$. We assume that all currency trade is ex ante voluntary; prior to drawing an allocation from a lottery any individual can say no to that lottery and quit currency exchange with what she brought in. However, if both individuals in a meeting say yes to a lottery, then they go along with any outcome of that lottery even if that

¹⁴See Berentsen, Molico, and Wright (2002) for a treatment of lotteries in a random matching model of money.

outcome is unfavorable ex post.

Once tourists exchange their money, they enter foreign country, where they meet foreign sellers with probability one. Because the post-trade allocation of currency is an outcome of a lottery over set \mathbb{S} , some individuals may enter foreign country having no money. Such individuals can neither consume (because they have no money to spend) nor produce (because we assume that foreigners are not allowed to work).

The timing of events is the following. Individuals start a period having either no money or one unit of domestic money. Then, each individual gets a realization of the preference shock. After that current prices \mathbf{p} are observed by all individuals. Then, those of the buyers who like foreign goods proceed to the currency exchange and trade domestic money for foreign money (that step includes bargaining about lotteries under floating rates), buy foreign goods (if foreign money is acquired as a result of trade in the currency exchange), consume, return to their home country (having either no money or a unit of domestic money if domestic money was retained as a result of trade in the currency exchange), and move to the next period. Those of the buyers who like domestic goods, meet their compatriots, trade (provided that their meeting is a trade meeting), consume, and move to the next period.

3 Fixed exchange-rate regime: the value of money

We start with exposition of the fixed exchange-rate regime. As we already said, the distinction between fixed and floating exchange-rate regimes is the distinction between who sets up lotteries in the currency exchange. Under fixed exchange-rate regime there is a third party referred to as the government, whose job is to pick a lottery and prevent individuals from defection to other lotteries; under floating exchange-rate regime such a party is absent, so that individuals bargain about lotteries and then follow outcomes provided by the agreed upon lotteries.

Given a lottery τ and the distribution μ of prices, the value of money is expected discounted utility from future consumption. We assume that in the end of a period all of the stock of money i is held in country i . Then, in the next period, citizens of country i carry a measure $\gamma(1 - \tau_i)$ of money i to country $-i$ as a result of allocation implied by the lottery τ . By the end

of that period all of these people return to their home country and, because they cannot spend domestic money abroad, all of that money travels back to country i , so that all-domestic-money-in-domestic-country distribution of money is a stationary distribution. Given that end-of-period distribution, let V_{ki} be the end-of-period value of having k , $k \in \{0, i\}$, units of money i by a citizen of country i , and let $\mathbf{V}_i \equiv (V_{0i} V_{ii})$. The value function \mathbf{V}_i , which we refer to as the *future value of money i* , is a solution to the following two-equation system of Bellman equations:

$$\mathbf{V}'_i = \beta [\mathbf{Q}'_i + T_i \mathbf{V}'_i], \quad (1)$$

where \mathbf{Q}_i is the vector of next-period expected gains from trade and T_i is the transition matrix for money holdings.

Because sellers never travel abroad, the expected payoff of a seller in a period is:

$$-(\psi_i + \gamma\tau_i) \int \frac{1}{p_i} d\mu,$$

where $\psi_i \equiv \frac{m_i - \gamma}{N}$ is the probability to meet an appropriate domestic buyer and $\gamma\tau_i$ is the probability to meet an appropriate foreign buyer. If a buyer likes domestic goods (in which case she stays in home country), then her expected period payoff is:

$$\varphi_i \int u \left(\frac{1}{p_i} \right) d\mu,$$

where $\varphi_i \equiv \frac{1 - m_i}{N}$ is the probability to meet an appropriate seller. If a buyer likes foreign goods (in which case she travels to a foreign country), then her expected period payoff is:

$$\tau_{-i} \int u \left(\frac{1}{p_{-i}} \right) d\mu,$$

which is simply the expected utility of consumption abroad times the probability of getting foreign money in the currency exchange. Then,

$$\mathbf{Q}'_i = \begin{bmatrix} -(\psi_i + \gamma\tau_i) \int \frac{1}{p_i} d\mu \\ \left(1 - \frac{\gamma}{m_i}\right) \varphi_i \int u \left(\frac{1}{p_i} \right) d\mu + \tau_{-i} \frac{\gamma}{m_i} \int u \left(\frac{1}{p_{-i}} \right) d\mu \end{bmatrix}, \quad (2)$$

where $\frac{\gamma}{m_i}$ is the probability that domestic buyer travels abroad.

We require that in equilibrium there is a trade in every trade meeting, so that the transition matrix T_i is:

$$T_i = \begin{bmatrix} 1 - (\psi_i + \gamma\tau_i) & \psi_i + \gamma\tau_i \\ \left(1 - \frac{\gamma}{m_i}\right)\varphi_i + \frac{\gamma}{m_i}\tau_i & \left(1 - \frac{\gamma}{m_i}\right)(1 - \varphi_i) + \frac{\gamma}{m_i}(1 - \tau_i) \end{bmatrix}. \quad (3)$$

Notice that because foreign money (if acquired) is spent with probability one within the same period, the probability of its acquisition, τ_{-i} , does not appear in the period-to-period transition matrix T_i ; acquisition of foreign money yields an expected payoff:

$$\tau_{-i} \int u\left(\frac{1}{p_{-i}}\right) d\mu,$$

which shows up in the vector of one-period expected gains from trade, \mathbf{Q}_i .

Because the mapping $G(\mathbf{x}') = \beta[\mathbf{Q}'_i + T_i\mathbf{x}'_i]$ is a contraction, system (1) has a unique solution:

$$\mathbf{V}'_i = \left(\frac{1}{\beta}I - T_i\right)^{-1} \mathbf{Q}'_i, \quad (4)$$

where I is a 2×2 identity matrix.

The future value of money \mathbf{V}_i is the continuation value of using money i as a medium of exchange. However, our sequence of events implies that agents trade knowing both whether they like domestic or foreign goods and current prices. Consequently, their trade depends on the *current value* rather than future value of money. Given domestic price level p_i , let $v_{kii}(p_i)$ denote the value of k , $k \in \{0, i\}$, units of money i held by domestic resident, who likes domestic goods (stays in country i that period). Then, the current value of a unit of money i held by such an individual, $\mathbf{v}_{ii}(p_i) \equiv (v_{0ii}(p_i) \ v_{i ii}(p_i))$, satisfies:

$$\mathbf{v}'_{ii}(p_i) = \beta[\mathbf{R}'_{ii}(p_i) + T_{ii}\mathbf{V}'_i], \quad (5)$$

where $\mathbf{R}_{ii}(p_i)$ is the vector of one-period gains from trade and T_{ii} is the corresponding transition matrix for money:

$$\mathbf{R}_{ii}(p_i) = \begin{bmatrix} -(\psi_i + \gamma\tau_i) \frac{1}{p_i} & \varphi_i u\left(\frac{1}{p_i}\right) \end{bmatrix} \quad (6)$$

and

$$T_{ii} = \begin{bmatrix} 1 - (\psi_i + \gamma\tau_i) & \psi_i + \gamma\tau_i \\ \varphi_i & 1 - \varphi_i \end{bmatrix}. \quad (7)$$

Likewise, the current value, $\mathbf{v}_{i-i}(p_{-i})$, of a unit of money i held by domestic citizen, who likes foreign goods, is the solution to:

$$\mathbf{v}'_{i-i}(p_{-i}) = \beta [\mathbf{R}'_{i-i}(p_{-i}) + T_{i-i}\mathbf{V}'_i],$$

where

$$\mathbf{R}_{i-i}(p_{-i}) = \begin{bmatrix} 0 & \tau_{-i}u \left(\frac{1}{p_{-i}} \right) \end{bmatrix} \quad (8)$$

and

$$T_{i-i} = \begin{bmatrix} 1 & 0 \\ \tau_i & 1 - \tau_i \end{bmatrix}. \quad (9)$$

The matrix T_{i-i} is the transition matrix associated with the transfer of money in currency exchange.

An individual is willing to trade if trade is not worse than doing nothing in a meeting. For those, who like domestic goods, the latter yields two incentive compatibility constraints:

$$\mathbf{v}_{ii}(p_i) \geq \beta\mathbf{V}_i,$$

which simplify to a familiar double inequality:

$$\frac{1}{p_i} \leq V_{ii} - V_{0i} \leq u \left(\frac{1}{p_i} \right). \quad (10)$$

The meaning of (10) is that quantity $q_i \equiv \frac{1}{p_i}$, produced in domestic trade meetings, must be in some middle range, so that sellers are willing to produce that quantity in exchange for money and buyers are willing to part with money in exchange for quantity q_i of consumption good. The incentive compatibility constraints for those who like foreign goods:

$$\mathbf{v}_{i-i}(p_{-i}) \geq \beta\mathbf{V}_i,$$

which yields a single non-trivial inequality:

$$\tau_{-i}u \left(\frac{1}{p_{-i}} \right) - \tau_i (V_{ii} - V_{0i}) \geq 0, \quad (11)$$

the meaning of which is that the expected gain from trade in the currency exchange is nonnegative.

We can now give the following definition.

Definition 1. *An allocation (μ, τ) is said to be incentive feasible if, given the distribution μ and the lottery τ , the implied value function \mathbf{V}_i satisfies all incentive compatibility constraints in (10) and (11) for all \mathbf{p} in the support of μ . We say that an allocation (μ, τ) is interior if it is incentive feasible and for all \mathbf{p} in the support of μ all incentive compatibility constraints in (10) and (11) are slack.*

Because we require all incentive compatibility constraints hold for any price draw \mathbf{p} in the support of μ , Definition 1 is consistent with ex post individual rationality in goods-for-money trade meetings, but not in the currency exchange meetings. In currency exchange meetings individuals go along even with ex post unfavorable allocations of money implied by the lottery τ . The requirement we impose on incentive feasible lotteries is ex ante individual rationality; because money is indivisible, that weaker requirement is necessary for interior lotteries to emerge in equilibrium (see Berentsen, Molico, and Wright, 2002, for further details).

4 Floating exchange-rate regime

Under floating exchange-rate regime there is no government to set up lotteries in the currency exchange; this task is performed by individuals. We assume that before individuals “throw the dice” and follow the supplied post-trade allocation of money, they engage in bilateral bargaining about the lottery to which they tune “the dice”. Once an agreement in bargaining is reached, the pair sets up the agreed upon lottery, draws an outcome, and follows the provided allocation of money. We adopt the bargaining solution instead of e.g. competitive outcomes because randomized trades of indivisible objects give rise to a multiplicity of walrasian equilibria. While this would add to exchange-rate indeterminacy under floating rates, such an indeterminacy would be a consequence of indivisibility of money.

The bargaining game we assume is a version of the alternating-offer bargaining game of Rubinstein (1982) and Trejos and Wright (1995). An important difference of our game is that waiting changes outside options of

the bargaining parties. One consequence of that is that the bargaining solution here does not coincide with the generalized Nash bargaining solution as it does in the static game. Bargaining solution when the outside option is changing is considered in Ennis (2001). Because here price draws \mathbf{p} are *i.i.d.*, our problem is simpler than that studied by Ennis; however, that relative simplicity allows us to increase the state space of the problem and to consider uncertainty over continuous sets of prices.

4.1 The bargaining game

The game is the following. Individuals make offers about the lotteries; each of the two individuals in a meeting is equally likely to make an offer. The partner can either accept an offer, or reject an offer but stay for the next bargaining round, or quit bargaining. Acceptance of an offer ends bargaining; the two individuals draw an outcome from the agreed upon lottery and follow the provided allocation of money. Rejection of an offer implies no action in a meeting; the pair simply proceeds to the next bargaining round. Quitting ends bargaining; nothing happens in a meeting and the two individuals quit currency exchange. We introduce an option to quit into an otherwise standard bargaining model to make bargaining voluntary. Facing being locked in bargaining for a potentially long time, individuals may accept unfavorable offers just to quit currency exchange and continue with their options in home country. Quitting permits individuals to reject extreme prices and keep their money to buy consumption at home.

We allow for one bargaining round and one price draw per period; the latter is assumed to make the sequence of actions under floating rates be consistent with that under fixed rates. Given that prices change once in a period, letting for more than one bargaining round in a period does not change main results of the paper, but complicates exposition. Notice that a pair in the currency exchange can bargain for many periods; later we impose conditions which guarantee that there are no agreement delays in equilibrium. Individuals, who leave currency exchange, trade with foreign sellers at current prices \mathbf{p} , so that there is no price discrimination of foreigners. Recall, that in goods-for-money trades individuals do not bargain but go along with any ex post individually rational outcome.

For the sake of exposition we describe the one-shot bargaining game first. Given \mathbf{p} and the future value of money \mathbf{V}_i , those who bargain over lotteries,

seek to maximize the (net) current value of money they have:

$$\tau_{-i} u \left(\frac{1}{p_{-i}} \right) - \tau_i (V_{ii} - V_{0i}). \quad (12)$$

Because the objective is linear, for any given value of money in (12) there are many lotteries, among which individuals are indifferent. That is why making offers about pairs (τ_A, τ_B) yields a multiplicity of solutions. To circumvent the problem it is convenient to assume that agents bargain about the current value of money implied by lotteries (as opposed to bargaining about lotteries) and accept any lottery which yields the desired payoff. Given reservation payoff π_{-i} of a foreigner, the lottery, which yields the highest payoff to a citizen of country i is a solution to:

$$\Xi_i(\pi_{-i}; \mathbf{p}) \equiv \max_{\tau} \left[\tau_{-i} u \left(\frac{1}{p_{-i}} \right) - \tau_i (V_{ii} - V_{0i}) \right]$$

subject to

$$\tau_i u \left(\frac{1}{p_i} \right) - \tau_{-i} (V_{-i-i} - V_{0-i}) = \pi_{-i}.$$

The payoff of agent i is, then:

$$\Xi_i(\pi_{-i}; \mathbf{p}) = \begin{cases} u \left(\frac{1}{p_{-i}} \right) - (V_{ii} - V_{0i}) \frac{\pi_{-i} + V_{-i-i} - V_{0-i}}{u \left(\frac{1}{p_i} \right)} & \text{if } 0 \leq \pi_{-i} \leq \pi_{-i}^* \\ u \left(\frac{1}{p_{-i}} \right) \frac{u \left(\frac{1}{p_i} \right) - \pi_{-i}}{V_{-i-i} - V_{0-i}} - (V_{ii} - V_{0i}) & \text{if } \pi_{-i}^* \leq \pi_{-i} \leq \bar{\pi}_{-i} \end{cases}, \quad (13)$$

where

$$\pi_{-i}^* = u \left(\frac{1}{p_i} \right) - (V_{-i-i} - V_{0-i}),$$

and $\bar{\pi}_{-i}$ is the maximum payoff of the opponent that agent i goes along with. The function $\Xi_i(\pi_{-i}; \mathbf{p})$ is the Pareto frontier of agent i ; it is a piecewise-linear, concave, and strictly decreasing function. Because in the one-shot game there is no option to continue bargaining in the next round, the best strategy of a person who makes an offer is to appropriate all of the surplus by making her partner be indifferent between acceptance and quitting. For the rest of this section we assume that citizen of country i makes an offer and her opponent $-i$ responds. Then individual i claims the maximum payoff $\bar{\pi}_i$, which is a unique solution to:

$$\Xi_{-i}(\bar{\pi}_i; \mathbf{p}) = 0.^{15}$$

Because in equilibrium any solution to the bargaining game must be incentive feasible, (10) implies that $\bar{\pi}_i$ is non-negative, so that the one-shot game always entails an agreement.

However, in an infinitely repeated game, individuals may prefer to wait for better terms of trade, especially if waiting incurs a small cost (e.g. if individuals are patient enough). Let Z_i denote the continuation value of the bargaining game for individual i , let δ_{-i} be the probability that the opponent accepts π_i , and ϕ_{-i} be the probability that the opponent quits bargaining. The Bellman equation of individual i is:

$$W_i(\mathbf{p}) = \beta \max_{\pi_i} [\delta_{-i}(\pi_i + V_{ii}) + \phi_{-i}V_{ii} + (1 - \delta_{-i} - \phi_{-i})Z_i]; \quad (15)$$

and the Bellman equation of her opponent $-i$ is:

$$w_{-i}(\mathbf{p}) = \beta \max [\Xi_{-i}(\pi_i; \mathbf{p}) + V_{-i-i}, V_{-i-i}, Z_{-i}]. \quad (16)$$

Here the optimal decision of individual $-i$ is to accept offers π_i below some threshold and to reject all offers above that threshold. Given that, the best strategy of agent i in (15) is to claim π_i , which makes $-i$ be indifferent between acceptance of that offer and the best of the two remaining alternatives. Thus, given continuation values Z_A and Z_B , the solution (π_A, π_B) to the bargaining game satisfies the following two-equation system:

$$\Xi_{-i}(\pi_i; \mathbf{p}) + V_{-i-i} = \max [V_{-i-i}, Z_{-i}].$$

Because both Pareto frontiers Ξ_A and Ξ_B are strictly decreasing, the solution is unique. To find the solution, we are only left to compute the continuation value of the bargaining game.

4.2 The continuation value of bargaining

The continuation value Z_i is the expected discounted stream of payoffs from future rounds. Because individuals do not observe future prices, but only know that these prices are independently drawn from a known distribution, the continuation value Z_i is the value of bargaining game where agents maximize the expectation of the current value of money. Because future price draws are identically and independently distributed, waiting does not change outside options of the bargaining parties, so that the continuation game is a static game. The one-shot objective of an individual is:

$$\widehat{\Xi}_i(\pi_{-i}) \equiv \max_{\tau} \left[\tau_{-i} \int u \left(\frac{1}{p_{-i}} \right) d\mu - \tau_i (V_{ii} - V_{0i}) \right], \quad (17)$$

subject to:

$$\tau_i \int u \left(\frac{1}{p_i} \right) d\mu - \tau_{-i} (V_{-i-i} - V_{0-i}) = \pi_{-i},$$

which yields a Pareto frontier $\widehat{\Xi}_i(\pi_{-i})$, which differs from (13) only in that the current prices are replaced by their expectation:

$$\widehat{\Xi}_i(\pi_{-i}) = \begin{cases} \int u \left(\frac{1}{p_{-i}} \right) d\mu - (V_{ii} - V_{0i}) \frac{\pi_{-i} + V_{-i-i} - V_{0-i}}{\int u \left(\frac{1}{p_i} \right) d\mu} & \text{if } 0 \leq \pi_{-i} \leq \widehat{\pi}_{-i}^* \\ \int u \left(\frac{1}{p_{-i}} \right) d\mu \frac{\int u \left(\frac{1}{p_i} \right) d\mu - \pi_{-i}}{V_{-i-i} - V_{0-i}} - (V_{ii} - V_{0i}) & \text{if } \widehat{\pi}_{-i}^* \leq \pi_{-i} \end{cases}, \quad (18)$$

where $\widehat{\pi}_{-i}^* \equiv \int u \left(\frac{1}{p_i} \right) d\mu - (V_{-i-i} - V_{0-i})$.

Because each individual in a meeting is equally likely to make an offer, the continuation value Z_i is:

$$Z_i = \frac{1}{2} [W_i + w_i],$$

where W_i and w_i solve the following system of Bellman equations:

$$W_i = \beta \max_{\pi_i} [\delta_{-i} (\pi_i + V_{ii}) + \phi_{-i} V_{ii} + (1 - \delta_{-i} - \phi_{-i}) Z_i] \quad (19)$$

and

$$w_{-i} = \beta \max [\widehat{\Xi}_{-i}(\pi_i) + V_{-i-i}, V_{-i-i}, Z_{-i}]. \quad (20)$$

As above, the optimal strategy of individual i is to make the opponent be indifferent between acceptance and the best among rejection and quitting, which yields:

$$w_{-i} = \beta \left(\widehat{\Xi}_{-i}(\pi_i) + V_{-i-i} \right) = \beta \max [V_{-i-i}, Z_{-i}]. \quad (21)$$

We consider equilibria in which individuals never quit bargaining.¹⁶ No-quitting implies that the continuation value of the bargaining game must be at least as large as the value of quitting,

$$Z_{-i} \geq V_{-i-i}. \quad (22)$$

¹⁶Equilibria in which individuals quit bargaining in the continuation game are equilibria of the one-shot game.

Provided that conditions (22) are satisfied, the continuation game has many Nash equilibria. However, one can show (by application of the implicit function theorem) that threats to delay agreement by those who respond to offers, reduce their payoffs and, hence, are not credible. In other words, the only subgame perfect equilibrium here is one, where agreement is immediate, which is a standard result for that class of games.¹⁷

Because waiting is costly, the party who makes an offer has an advantage of moving first and can claim a higher payoff for herself. The Bellman equations (19)-(20) can be written as:

$$W_i = \beta (\pi_i + V_{ii}) \quad (23)$$

and

$$w_i = \beta Z_i, \quad (24)$$

so that:

$$\left(\frac{2}{\beta} - 1\right) Z_i = \pi_i + V_{ii}. \quad (25)$$

Then, it follows from (21) that the optimal payoff of individual i can be obtained from:

$$\widehat{\Xi}_{-i}(\pi_i) + V_{-i-i} = Z_{-i}, \quad (26)$$

where because individual i makes an offer, $\pi_i \geq \widehat{\pi}_i^*$, so that:

$$\widehat{\Xi}_{-i}(\pi_i) = \int u\left(\frac{1}{p_i}\right) d\mu \frac{\int u\left(\frac{1}{p_{-i}}\right) d\mu - \pi_i}{V_{ii} - V_{0i}} - (V_{-i-i} - V_{0-i}). \quad (27)$$

Then, combining (25)-(27), one obtains that the continuation values Z_A and Z_B satisfy the following two-equation system:

$$\left(\frac{2}{\beta} - 1\right) Z_i + \frac{V_{ii} - V_{0i}}{\int u\left(\frac{1}{p_i}\right) d\mu} (Z_{-i} - V_{0-i}) = \int u\left(\frac{1}{p_{-i}}\right) d\mu + V_{ii}. \quad (28)$$

Because in equilibrium any solution to the continuation game must be incentive feasible, the determinant Δ of (28) is positive:

$$\Delta \geq \left(\frac{2}{\beta} - 1\right)^2 - 1 > 0,$$

¹⁷See Rubinstein (1982) for further discussion.

so that (28) has a unique solution. The solution $(\widehat{Z}_A, \widehat{Z}_B)$ is strictly increasing in β and:

$$\lim_{\beta \rightarrow 1} \widehat{Z}_i = \int u \left(\frac{1}{p_{-i}} \right) d\mu + V_{0i}. \quad (29)$$

Then, the bargaining solution of the continuation game is:

$$\widehat{\tau}_i = \frac{\widehat{Z}_{-i} - V_{0-i}}{\int u \left(\frac{1}{p_i} \right) d\mu} < 1 \quad (30)$$

and

$$\widehat{\tau}_{-i} = 1.$$

As we already said, individual i who moves first has an advantage, so that she claims her opponent's money with probability one and surrenders her money with probability less than one. In the limit, as $\beta \rightarrow 1$, $\widehat{\tau}_i \rightarrow 1$; so that if waiting becomes costless, then the two individuals simply swap their money.

4.3 Equilibria

We now turn to the description of equilibria. Without any changes to substance we assume that in all meetings citizens of only one country i make offers in a given bargaining round, which allows to avoid the inconvenience of having to deal with two distinct lotteries (and hence exchange rates) being offered at the same time. Given the continuation value of a foreigner and the price draw \mathbf{p} , the payoff domestic citizen attains for herself, $\pi_i(\mathbf{p})$, is a unique solution to:

$$\Xi_{-i}(\pi_i; \mathbf{p}) + V_{-i-i} = \widehat{Z}_{-i}. \quad (31)$$

Solving (31) for the payoff $\pi_i(\mathbf{p})$, one obtains:

$$\pi_i(\mathbf{p}) = \begin{cases} \pi_i^* - u \left(\frac{1}{p_{-i}} \right) \frac{\widehat{\tau}_i \int u \left(\frac{1}{p_i} \right) d\mu - u \left(\frac{1}{p_i} \right)}{V_{-i-i} - V_{0-i}} & \text{if } \widehat{\tau}_i \int u \left(\frac{1}{p_i} \right) d\mu \geq u \left(\frac{1}{p_i} \right) \\ \pi_i^* - (V_{ii} - V_{0i}) \frac{\widehat{\tau}_i \int u \left(\frac{1}{p_i} \right) d\mu - u \left(\frac{1}{p_i} \right)}{u \left(\frac{1}{p_i} \right)} & \text{if } \widehat{\tau}_i \int u \left(\frac{1}{p_i} \right) d\mu \leq u \left(\frac{1}{p_i} \right) \end{cases}. \quad (32)$$

Given current prices \mathbf{p} , the payoff $\pi_i(\mathbf{p})$ has to be at least as large as the continuation value of bargaining \widehat{Z}_i , otherwise an individual i will wait for

the next bargaining round. If the price p_i is high, i.e. if:

$$\hat{\tau}_i \int u\left(\frac{1}{p_i}\right) d\mu \geq u\left(\frac{1}{p_i}\right), \quad (33)$$

then the current value of money i is low, which means that it is easier to part with that money. In that case provided that prices \mathbf{p} satisfy:

$$1 - \frac{\hat{\tau}_i \int u\left(\frac{1}{p_i}\right) d\mu - u\left(\frac{1}{p_i}\right)}{V_{-i-i} - V_{0-i}} \geq \frac{\hat{\tau}_{-i} \int u\left(\frac{1}{p_{-i}}\right) d\mu}{u\left(\frac{1}{p_{-i}}\right)}, \quad (34)$$

agent i demands the following lottery:

$$\tau_i = 1, \quad (35)$$

and

$$\tau_{-i} = 1 - \frac{\hat{\tau}_i \int u\left(\frac{1}{p_i}\right) d\mu - u\left(\frac{1}{p_i}\right)}{V_{-i-i} - V_{0-i}}, \quad (36)$$

so that she gives out her money for sure and gets money $-i$ with probability less than one. Condition (34) implies that if both prices p_A and p_B satisfy (33), then an agreement is not reached in that round and the bargaining parties will wait hoping to get better terms in the next round.

If the current price p_i is low, i.e.

$$\hat{\tau}_i \int u\left(\frac{1}{p_i}\right) d\mu \leq u\left(\frac{1}{p_i}\right), \quad (37)$$

then provided that prices \mathbf{p} satisfy:

$$\frac{\hat{\tau}_i \int u\left(\frac{1}{p_i}\right) d\mu}{u\left(\frac{1}{p_i}\right)} \leq 1 - \frac{\hat{\tau}_{-i} \int u\left(\frac{1}{p_{-i}}\right) d\mu - u\left(\frac{1}{p_{-i}}\right)}{V_{i-i} - V_{0i}}, \quad (38)$$

the citizen of country i claims money $-i$ and agrees to surrender her money with probability less than one:

$$\tau_{-i} = 1, \quad (39)$$

and

$$\tau_i = \frac{\hat{\tau}_i \int u\left(\frac{1}{p_i}\right) d\mu}{u\left(\frac{1}{p_i}\right)}. \quad (40)$$

In turn, condition (38) implies that if both prices p_A and p_B satisfy (37), then an agreement is reached immediately. If prices countermove, then given the bargaining solution (36) and (40), condition (34) means that individual i gets at least as much of foreign money as (given the same prices \mathbf{p}) she would get had it been the turn of her opponent to make an offer. Similarly, condition (38) means that agent i gives out no more of her money than she would give out had it been the turn of her opponent to make an offer. In both cases the meaning of inequalities (34) and (38) is that an agreement is reached if and only if there is no benefit of giving the opponent a chance to make an offer by waiting for the next bargaining round.

We say that a lottery τ is a *solution of the bargaining problem* if, given measure μ , the current draw of prices \mathbf{p} from that measure, and the future value of money \mathbf{V}_i , no-quitting conditions (22) hold and prices \mathbf{p} and lottery τ satisfy either (33)-(36) or (37)-(40). Notice that we call lottery τ a bargaining solution only if the underlying draw \mathbf{p} of current prices is consistent with immediate agreement in bargaining.

Given measure μ and the future value of money \mathbf{V}_i , let $S_{\mu, \mathbf{V}}$ be the set of prices consistent with immediate agreement in bargaining:

$$S_{\mu, \mathbf{V}} \equiv \{\mathbf{p} : (34), (38) \text{ hold}\}.$$

Because our definition of the future value of money assumes that by the end of every period all tourists return to their home countries, we consider measures μ consistent with immediate agreement in bargaining in all currency exchange meetings. In other words, we limit ourselves to measures μ such that $\text{supp } \mu \subset S_{\mu, \mathbf{V}}$. If $\beta < 1$, then any measure μ with sufficiently small support satisfies that requirement. The intuition is that if μ has a small support (i.e. the uncertainty with respect to division of the gains from trade is small), then the cost of waiting a period exceeds the benefit from potentially better terms of trade in the future. However, if waiting is costless, then (because the set $S_{\mu, \mathbf{V}}$ is convex) any non-degenerate measure fails to yield an immediate agreement in bargaining at all times. We now give the definition of a stationary equilibrium.

Definition 2. A pair, price draw \mathbf{p} and allocation $(\mu, \boldsymbol{\tau})$, is said to be an equilibrium point if *i)* given $(\mu, \boldsymbol{\tau})$ the value function \mathbf{V} is given by (4) and $(\mu, \boldsymbol{\tau})$ is incentive feasible, *ii)* given μ , \mathbf{V} and \mathbf{p} , the lottery $\boldsymbol{\tau}$ is a solution of the bargaining problem. We say that measure μ is a stationary equilibrium if for every \mathbf{p} in the support of μ , there exists a lottery $\boldsymbol{\tau}$ such that $(\mathbf{p}, (\mu, \boldsymbol{\tau}))$ is an equilibrium point.

Let d be the diameter of measure μ .¹⁸ Before we turn to existence, we find it useful to discuss the limiting case of $d = 0$ and $\beta = 1$. Even though as β approaches one, the future value of money \mathbf{V}_i becomes unbounded, the gains from being a buyer, $V_{ii} - V_{0i}$, remain finite. That allows us to define stationary equilibria for $\beta = 1$ as the limit of those with $\beta < 1$. Because by definition $d = 0$ implies that measure μ is degenerate,

$$\lim_{d \rightarrow 0} \max_{\mathbf{p} \in \text{supp } \mu} \left| u \left(\frac{1}{p_i} \right) - \int u \left(\frac{1}{p_i} \right) d\mu \right| = 0.$$

Because $\lim_{\beta \rightarrow 1} \widehat{\tau}_i = 1$, it follows from (36) and (40) that the bargaining solution with $\beta = 1$ and $d = 0$ is the unit lottery: $\tau_A = \tau_B = 1$.

In other words, with $d = 0$ and $\beta = 1$ any incentive feasible allocation (δ, Υ) , where δ is degenerate and Υ is the unit lottery, is an equilibrium point and, because the support of δ is a singleton, is a stationary equilibrium. That, as well as continuity of the bargaining solution and of the future value of money \mathbf{V}_i , allows to develop a local existence argument based on application of the Brouwer's fixed point theorem. By saying that our argument is local we mean that it asserts existence of an interval $(\beta^*, 1]$ such that for any $\beta \in (\beta^*, 1]$ there is a non-degenerate measure μ with sufficiently small support which satisfies Definition 2.

The sketch of the argument is as follows. Given $\beta = 1$, let us fix some interior allocation (δ, Υ) , called a *fixed-price allocation*. Because (δ, Υ) is interior, one can find some β close to 1, a compact and convex neighborhood \mathcal{U} of the unit lottery Υ , and a non-degenerate measure μ with a small enough support such that an allocation $(\mu, \boldsymbol{\tau})$ is incentive feasible for every lottery $\boldsymbol{\tau} \in \mathcal{U}$, no-quitting (22) is satisfied, and $\text{supp } \mu \subset S_{\mu, \mathbf{V}}$. Next, define a map $\Psi_\mu(\boldsymbol{\tau}; \mathbf{p})$ from \mathcal{U} into $[0, 1]^2$ as follows. Given measure μ and a price draw $\mathbf{p} \in \text{supp } \mu$, pick a lottery $\boldsymbol{\tau}$ from \mathcal{U} and use (4) to compute the future

¹⁸By definition, the diameter of a measure is the maximum distance between any two points in its support.

value of money \mathbf{V}_i . Then, given \mathbf{p} , μ , and \mathbf{V}_i , use (35)-(36), (39)-(40) to compute the solution to the bargaining problem and let that solution be the image of τ under $\Psi_\mu(\tau; \mathbf{p})$. Because β is close to 1 and the support of μ is small, the image $\Psi_\mu(\tau; \mathbf{p})$ of τ is close to the unit lottery Υ . If for every $\mathbf{p} \in \text{supp } \mu$ and $\tau \in \mathcal{U}$, $\Psi_\mu(\tau; \mathbf{p}) \in \mathcal{U}$, then because the map Ψ_μ is continuous, Brouwer's theorem applies and we are done. If there is a pair \mathbf{p} and τ such that $\Psi_\mu(\tau; \mathbf{p}) \notin \mathcal{U}$, then we take a higher value of β and another measure μ with a smaller support. Because as $\beta \rightarrow 1$ and $d \rightarrow 0$ the limit of the bargaining solution (35)-(36), (39)-(40) is the unit lottery Υ , eventually the image of τ under $\Psi_\mu(\tau; \mathbf{p})$ is in \mathcal{U} for all $\mathbf{p} \in \text{supp } \mu$ and $\tau \in \mathcal{U}$, so that Definition 2 is satisfied.

Proposition 1. *Let $\beta = 1$ and let (δ, Υ) be an arbitrary interior fixed-price allocation. Then, there exists a critical value $\beta^* < 1$ such that for every $\beta \in (\beta^*, 1)$ there exists a nondegenerate measure μ which is a stationary equilibrium.*

An immediate implication of Proposition 1 is exchange-rate disconnect puzzle. Non-degeneracy of equilibrium measure μ implies that with positive probability prices drawn from μ are not average prices. Then, the bargaining solution implies that equilibrium exchange rates are stochastic. Because all model parameters remain unchanged, exchange-rate movements occur vis-a-vis constant fundamentals.¹⁹ Also, because all individuals face uniform price draw \mathbf{p} at any particular date, the realized equilibrium exchange rate τ is the same across all currency exchange meetings. That is, there is no cross-section difference in exchange rates in equilibrium. However, because in every period the economy is hit by different realizations of the shock, there are time-series movements of exchange rates.

Even though Proposition 1 contains a local result, it is consistent with excess volatility puzzle. Examples, where exchange-rate volatility exceeds CPI volatility many times are straightforward (see Appendix 6.1). Roughly, such examples are consistent with sufficient patience and sufficiently high marginal valuation of consumption by individuals. In that case the model predicts substantial volatility of nominal exchange rates vis-a-vis almost constant CPI (and fixed fundamentals). That implies that there is little difference in time-series dynamics between nominal and real exchange rates; both of the two are disconnected from the real economy and both are much more

¹⁹Because money is indivisible, output fluctuates to the extent of small shocks to prices.

volatile then observable real variables such as output or money supplies, which is consistent with empirical evidence (see e.g. Obstfeld and Rogoff, 1996).

Given floating rates, a switch from floating to fixed exchange-rate regime (which is roughly equivalent to replacement of non-degenerate measure μ by a degenerate measure δ , see Proposition 1) has no effects other than reduction in exchange-rate volatility. Following a switch in exchange-rate regime, an outside observer will see a decrease in real exchange-rate volatility and no change in volatility of fundamentals. Under conditions discussed in Appendix 6.1 (patient individuals and high marginal valuation of consumption) such a reduction can be quite sharp and may, indeed, seem puzzling. A switch from fixed to floating rates (a replacement of degenerate measure δ by a non-degenerate measure μ) results in increase in nominal and real exchange-rate volatility vis-a-vis no change to volatility of fundamentals. That kind of exchange-rate behavior is consistent with exchange-rate behavior empirically documented by Flood and Rose (1995, p. 4), who write:

Most models of exchange rate determination argue that this [exchange rate] volatility is merely transferred to other economic loci, i.e., there is “conservation of volatility”. ... We argue empirically that the volatility is not in fact transferred to some other part of the economy; it simply seems to vanish.

Here the consequences of a regime switch are exactly as those in the above passage — the volatility “seems” to vanish.

Because the volatility is not transferred elsewhere, fixed exchange rates carry little impediment to the conduct of policy, in particular, monetary policy. The monetary authorities in both countries are free to adjust domestic money supplies keeping exchange rate fixed. However, an increase (decrease) in the money supply in one country alters the value of that country’s money, which provided that the money supply shock is large, may be inconsistent with willingness of individuals to exchange currencies at a posted rate. In that case there is a need to devalue (revalue); our model is therefore consistent with conventional wisdom that fixed exchange rates are hard to maintain in face of large shocks to fundamentals. Because individuals cannot hedge exchange-rate risks, fixing previously floating rates increases trade in the currency exchange and, hence, consumption and welfare. The latter, as well as (relative) policy freedom, implies that in our model fixed exchange rates

perform at least as good as floating exchange rates (see King, Wallace, Weber, 1992, for a similar result), which we are inclined to take more broadly as an argument in favor of fixed exchange rates.

5 Concluding remarks

The message of existing exchange-rate models is that the theory fails to explain (or predict) the behavior of exchange rates. Models, which partially succeed, tend to assume things such as extreme substitutability among currencies, non-rational behavior of traders, or asymmetric information about fundamentals. In many models volatility of exchange rates is simply an amplified volatility of fundamentals.

Here exchange-rate dynamics is fueled by small shocks pertaining to the division of the gains from trade. We show that a combination of search and legal restrictions can produce exchange-rate dynamics consistent with main empirical exchange-rate facts. The model is consistent with the following. Both nominal and real exchange rates are disconnected from the real economy, both are much more volatile than real variables such as output or money supplies, both can exhibit very similar behavior, and both fail to satisfy conservation of volatility.

In the model we abstract from production and capital markets. We surmise that bringing capital (asset) markets into the model does not change main results provided that there are search frictions in these markets similar to frictions in goods markets. The role of frictions in all markets is to attenuate the pass-through from exchange rates to goods prices and interest rates and thus to disconnect exchange rates from fundamentals by weakening standard no-arbitrage arguments such as PPP and UIP. Our combination of frictions is rather different from that in Devereux and Engel (2002), who assume local currency pricing, heterogeneous product distribution in goods markets, and consistently biased expectations in international asset markets.

There are some exchange-rate facts, which we cannot explain here; in particular, our model cannot account for the systematic bias in currency forward rates and for observed persistence in exchange-rate time series. One way to deal with forward bias puzzle is to allow for contracts, which guarantee particular lotteries to be delivered at future dates. Because exchange rates in our model can be extremely volatile, spot prices of such contracts would include a premium for large exchange-rate risks, making model predictions

be consistent with the evidence.

As regards persistence, here exchange-rate dynamics is driven by shocks to current prices whereas the future value of money remains constant over time. A more intuitive source of exchange-rate indeterminacy is uncertainty about the future value of money. Such a model would be capable of generating sufficient persistence in exchange-rate time series (which is observed in the data), which we do not have because in our setting future prices are *i.i.d.* Although a model, which builds upon uncertainty about future value of money, is easy to articulate, it is less easy to solve. In particular, relative simplicity of our bargaining solution depends on future prices being *i.i.d.* Then, generalization of results in Ennis (2001) to the case of large state spaces seems to be an important step towards persistent exchange rates.

6 Appendix

6.1 Numerical example

Here we construct an example, which demonstrates that volatility of nominal exchange rates can be much larger than price volatility. Because volatility of nominal exchange rate is the sum of real exchange-rate volatility and price volatility, the latter implies that real exchange rates can be as volatile as nominal exchange rates. The example is also a useful illustration of model implications about the effects of an exchange-rate regime switch; we obtain that qualitative behavior of exchange rates implied by the model is very much similar to that documented by Flood and Rose (1995).

Let us assume that the two countries are identical and that trade flows between them are negligibly small, i.e. $\gamma = 0$. Assume that utility function $u(x)$ is $u(x) = x^\alpha$, where $0 < \alpha < 1$. Let $q_i \equiv \frac{1}{p_i}$ be the quantity produced by producers in country i (in exchange for a unit of money) given current price p_i . Assume that the cumulative distribution of quantities is a uniform distribution over a square support,

$$\text{supp } \mu = \{(q_A, q_B) : q - d \leq q_A \leq q + d \text{ and } q - d \leq q_B \leq q + d\}$$

where q is the average quantity produced in both countries and d measures the extent of price uncertainty. One can use (4) to show that given that distribution:

$$\int u\left(\frac{1}{p_i}\right) d\mu = q^\alpha + O(d), \quad (41)$$

$$\int \frac{1}{p_i} d\mu = q, \quad (42)$$

and

$$V_{ii} - V_{0i} = \frac{(1 - m)q^\alpha + mq}{1 + rN} + O(d), \quad (43)$$

where m is money stock in both countries, $r \equiv \frac{1}{\beta} - 1$, is a period discount rate, and $O(d)$ satisfies $\lim_{d \rightarrow 0} O(d) = 0$.

As a measure of relative volatility of exchange rates and prices we choose the ratio of maximum percentage deviations of the two variables from their mean values. For prices, the maximum deviation from the mean is:

$$\Delta_p = \frac{d}{q} + O(d^2).$$

To obtain the maximum deviation of exchange rates we use the bargaining solution (36), which after substitution of (41)-(43) yields:

$$\Delta_\tau = (1 - \hat{\tau}_i) \frac{q^\alpha + O(d)}{\frac{(1-m)q^\alpha + mq}{1+rN}} + \frac{\alpha q^\alpha}{\frac{(1-m)q^\alpha + mq}{1+rN}} \frac{d}{q} + O(d^2).$$

Because the limit,

$$\lim_{\beta \rightarrow 1} \hat{\tau}_i = 1,$$

one can choose sufficiently small r , so that $1 - \hat{\tau}_i$ is of the same order as d^2 . The ratio $\frac{\Delta_\tau}{\Delta_p}$ is then:

$$\frac{\Delta_\tau}{\Delta_p} = (1 + rN) \frac{\alpha q^\alpha}{(1 - m)q^\alpha + mq} + O(d). \quad (44)$$

To show that the relative volatility can be large, assume the following parameters: $u(x) = x^{0.5}$, $q = 0.002$, $m = 0.5$, and $rN = 10$. These parameters depict a world where individuals are very patient, half of the population is endowed with money, and there is a large degree of specialization in production and consumption. As a consequence, individuals experience long runs of consumption opportunities, so that sustainable output is quite small and marginal valuation of consumption is large. One can verify that an allocation described by these parameters is incentive feasible and satisfies (22) provided that price volatility is small enough. Expression (44) evaluated at $d = 0$ yields $\frac{\Delta_\tau}{\Delta_p} \approx 10.5$, so that nominal exchange rate is ten times more volatile than prices.

That implies that real exchange rate is about ten times more volatile than prices and, hence, there is little difference in volatility between nominal and real exchange rates. High volatility of real exchange rates has been documented in many sources; a good review of such findings can be found in Obstfeld and Rogoff (1996) or in Alessandria (2003). The constructed example shows that in accord with empirical observations our model is consistent with highly volatile real exchange rates.

As regards volatility conservation, assume now that the government fixes nominal exchange rate. Then, real exchange-rate volatility is nothing but price volatility. Because price volatility is ten times smaller, an outside observer will see a sharp decline in real exchange-rate volatility. On the other hand, there will be no change in volatility of other macroeconomic variables — money supply as well as other real variables are fixed; given negligible

volume of trade between the two countries, output volatility is of the same order as price volatility and the latter is small. Similarly, a switch from fixed nominal exchange rates to floating, will produce large volatility of both nominal and real exchange rates without any changes to the volatility of other variables. This kind of exchange-rate behavior is documented in Flood and Rose (1995), Obstfeld and Rogoff (1996), etc.; our model is thus consistent with these empirical observations.

6.2 Proof of Proposition 1

Proposition 1. *Let $\beta = 1$ and let (δ, Υ) be an arbitrary interior fixed-price allocation. Then, there exists a critical value $\beta^* < 1$ such that for every $\beta \in (\beta^*, 1)$ there exists a nondegenerate measure μ which is a stationary equilibrium.*

Proof: First, let us define stationary equilibria for the case $\beta = 1$. As we already said, we define equilibria with $\beta = 1$ as the limit of those with $\beta < 1$. Observe that even though as β goes to 1 the future value of money \mathbf{V}_i becomes unbounded, the gain from being a buyer, $V_{ii} - V_{0i}$, remains finite:

$$\lim_{\beta \rightarrow 1} (V_{ii} - V_{0i}) = \frac{\left(1 - \frac{\gamma}{m_i}\right) \varphi_i \int u\left(\frac{1}{p_i}\right) d\mu + \tau_{-i} \frac{\gamma}{m_i} \int u\left(\frac{1}{p_{-i}}\right) d\mu + (\psi_i + \gamma \tau_i) \int \frac{1}{p_i} d\mu}{\left(1 - \frac{\gamma}{m_i}\right) \varphi_i + \frac{\gamma}{m_i} \tau_i + \psi_i + \gamma \tau_i},$$

where $\varphi_i \equiv \frac{1-m_i}{N}$ and $\psi_i \equiv \frac{m_i-\gamma}{N}$. Because the $\lim_{\beta \rightarrow 1} (V_{ii} - V_{0i})$ exists, one can define incentive feasible (interior) allocations for the case $\beta = 1$ in accord with Definition 1. Next observe that because:

$$\lim_{\beta \rightarrow 1} \widehat{Z}_i = \int u\left(\frac{1}{p_{-i}}\right) d\mu + V_{0i},$$

in the limit no-quitting conditions (22) can be written as:

$$\int u\left(\frac{1}{p_{-i}}\right) d\mu - (V_{ii} - V_{0i}) \geq 0, \quad (45)$$

where the gain from being a buyer, $V_{ii} - V_{0i}$, is given above. Also, it follows from (30) that the limit of the bargaining solution in the continuation game

is the unit lottery:

$$\lim_{\beta \rightarrow 1} \hat{\tau}_i = 1. \quad (46)$$

Further, recall that the set $S_{\mu, \mathbf{v}}$ of prices consistent with immediate agreement in bargaining is defined by the following two inequalities:

$$1 - \frac{\hat{\tau}_A \int u\left(\frac{1}{p_A}\right) d\mu - u\left(\frac{1}{p_A}\right)}{V_{BB} - V_{0B}} \geq \frac{\hat{\tau}_B \int u\left(\frac{1}{p_B}\right) d\mu}{u\left(\frac{1}{p_B}\right)} \quad (47)$$

and

$$\frac{\hat{\tau}_A \int u\left(\frac{1}{p_A}\right) d\mu}{u\left(\frac{1}{p_A}\right)} \leq 1 - \frac{\hat{\tau}_B \int u\left(\frac{1}{p_B}\right) d\mu - u\left(\frac{1}{p_B}\right)}{V_{AA} - V_{0A}}. \quad (48)$$

The set $S_{\mu, \mathbf{v}}$ is a convex set. Furthermore, if $\beta = 1$, then it follows from (46) that $\hat{\tau}_A = \hat{\tau}_B = 1$, so that convexity of $S_{\mu, \mathbf{v}}$ implies there does not exist a non-degenerate measure μ which satisfies $\text{supp } \mu \subset S_{\mu, \mathbf{v}}$. In other words, if waiting is costless, then there is no non-degenerate measure μ consistent with immediate agreement in currency exchange meetings for all realizations \mathbf{p} from that measure. Intuitively, this happens because if measure μ is non-degenerate, then with some positive probability prices drawn from μ yield poor gains from trade for at least one of the two individuals in a meeting. Because waiting is costless, then that person prefers to wait for a better price draw next period.

If $\beta < 1$, then μ can be non-degenerate and satisfy $\text{supp } \mu \subset S_{\mu, \mathbf{v}}$. The intuition here is that even though the gains from trade may be poor, it is better to agree to a higher price now than to wait for the next trade opportunity. However, as one would expect, the diameter of any measure μ which satisfies $\text{supp } \mu \subset S_{\mu, \mathbf{v}}$ must approach zero as $\beta \rightarrow 1$.

Given that any stationary equilibrium with $\beta = 1$ must be a fixed-price allocation, one can easily check that if $\beta = 1$, then any incentive feasible fixed-price allocation is a stationary equilibrium. If $\beta = 1$ and μ is degenerate, then the solution to the bargaining problem (36), (40) is the unit lottery. Then conditions (47), (48) hold at equality and no-quitting conditions (45) are implied by incentive feasibility constraints (11), so that Definition 2 is satisfied. We use that property of stationary equilibria with $\beta = 1$ to prove local existence of non-degenerate stationary equilibria for β sufficiently close to 1.

Let $\beta = 1$ and let (δ, Υ) be an interior fixed-price allocation. Let $\widehat{\mathbf{p}}$ be the unique point in support of degenerate measure δ , and let $u\left(\frac{1}{\widehat{p}_i}\right)$ and $\frac{1}{\widehat{p}_i}$ be the (expected) utility of consumption and the cost of production implied by δ . Then continuity of the future value of money (see 4) and of the bargaining solution in the continuation game (see 36 and 40) implies that there exists an interval $(\widetilde{\beta}, 1]$, a closed neighborhood \mathcal{U} of the unit lottery such that incentive compatibility constraints (10)-(11), no-quitting conditions (22) hold for any $\beta \in (\widetilde{\beta}, 1]$ and any allocation $(\delta, \boldsymbol{\tau})$, where $\boldsymbol{\tau} \in \mathcal{U}$. Without loss of generality we can take a circular neighborhood \mathcal{U} ,

$$\mathcal{U} = \{\boldsymbol{\tau} : \tau_A \leq 1, \tau_B \leq 1, \rho(\boldsymbol{\tau}, \Upsilon) \leq r_{\mathcal{U}}\},$$

where $r_{\mathcal{U}} > 0$ is the radius of \mathcal{U} and $\rho(x, y)$ is the standard distance function (metrics) in \mathbb{R}^2 .

Then, for every $\beta \in (\widetilde{\beta}, 1)$ consider a non-degenerate measure, denoted μ_{β} , which satisfies:

$$\int u\left(\frac{1}{p_i}\right) d\mu_{\beta} = u\left(\frac{1}{\widehat{p}_i}\right) \quad \text{and} \quad \int \frac{1}{p_i} d\mu_{\beta} = \frac{1}{\widehat{p}_i}. \quad (49)$$

It follows from (49) that the diameter of μ_{β} can be chosen small enough, so that no-delay conditions (47), (48) hold for any prices $\mathbf{p} \in \text{supp } \mu_{\beta}$. Without loss of generality we can choose the family of measures μ_{β} , so that the diameter of μ_{β} approaches zero as $\beta \rightarrow 1$.

Then, given an arbitrary price draw \mathbf{p} from the support of μ_{β} let us define a map $\Psi_{\mu_{\beta}}(\boldsymbol{\tau}; \mathbf{p})$, which maps \mathcal{U} into $[0, 1]^2$ as follows. Given μ_{β} , take some lottery $\boldsymbol{\tau} \in \mathcal{U}$ and compute the associated future value of money \mathbf{V} in accordance with (4). Then, given \mathbf{p} , μ_{β} and \mathbf{V} , compute the bargaining solution in accordance with (35)-(40) and let that solution be the image of $\boldsymbol{\tau}$ under $\Psi_{\mu_{\beta}}(\boldsymbol{\tau}; \mathbf{p})$. It follows from (4) and (35)-(40) that $\Psi_{\mu_{\beta}}(\boldsymbol{\tau}; \mathbf{p})$ is single-valued and continuous.

Then two cases are possible:

- i) the image of \mathcal{U} under $\Psi_{\mu_{\beta}}(\boldsymbol{\tau}; \mathbf{p})$ is a subset of \mathcal{U} for all \mathbf{p} in the support of μ_{β} ,
- ii) there is a pair $\mathbf{p} \in \text{supp } \mu_{\beta}$ and a lottery $\boldsymbol{\tau}$ such that the image of $\boldsymbol{\tau}$ under $\Psi_{\mu_{\beta}}(\boldsymbol{\tau}; \mathbf{p})$ is not in \mathcal{U} .

In the first case the Brouwer's fixed point theorem applies for all \mathbf{p} in the support of μ_β , so that in accordance with Definition 2, μ_β is a stationary equilibrium. Our goal is to prove that that is the case for all β sufficiently close to 1.

Given some β , $\beta \in (\tilde{\beta}, 1]$, and the associated measure μ_β , define $d(\beta)$ be the maximum distance between the unit lottery and the image of \mathcal{U} under $\Psi_{\mu_\beta}(\boldsymbol{\tau}; \mathbf{p})$,

$$d(\beta) = \max_{(\boldsymbol{\tau}, \mathbf{p}) \in \Gamma(\beta)} \rho \left(\Psi_{\mu_\beta}(\boldsymbol{\tau}; \mathbf{p}), \Upsilon \right),$$

where

$$\Gamma(\beta) \equiv \{ \boldsymbol{\tau}, \mathbf{p} : \boldsymbol{\tau} \in \mathcal{U}, \mathbf{p} \in \text{supp } \mu_\beta \}.$$

To show that measures μ_β are stationary equilibria for β sufficiently close to 1, it is sufficient to demonstrate that the limit (as β goes to 1) of distance $d(\beta)$ is zero.

Let us define the following three auxiliary functions:

$$G_1(\beta, \mathbf{p}) \equiv \max \left[0, \hat{\tau}_i \int u \left(\frac{1}{p_i} \right) d\mu_\beta - u \left(\frac{1}{p_i} \right) \right],$$

$$G_2(\beta, \mathbf{p}) \equiv \max \left[0, u \left(\frac{1}{p_i} \right) - \hat{\tau}_i \int u \left(\frac{1}{p_i} \right) d\mu_\beta \right],$$

$$F(\beta, \boldsymbol{\tau}) \equiv V_{-i-i} - V_{0-i},$$

where \mathbf{V}_{-i} is the future value of money $-i$ in (4) and $\hat{\tau}_i$ is the bargaining solution of the continuation game in (30).

Because the bargaining solution implies that one of the two currencies always changes hands with probability one,

$$d(\beta) = \max \left[\max_{(\boldsymbol{\tau}, \mathbf{p}) \in \Gamma(\beta)} \frac{G_1(\beta, \mathbf{p})}{F(\beta, \boldsymbol{\tau})}, \max_{(\boldsymbol{\tau}, \mathbf{p}) \in \Gamma(\beta)} \frac{G_2(\beta, \mathbf{p})}{u \left(\frac{1}{p_i} \right)} \right]. \quad (50)$$

Because μ_β satisfies (49) by construction, auxiliary functions G_1 , G_2 , and F are continuous functions of their arguments. Furthermore,

$$0 \leq d(\beta) \leq \max \left[\frac{\max_{(\boldsymbol{\tau}, \mathbf{p}) \in \Gamma(\beta)} G_1(\beta, \mathbf{p})}{\min_{(\boldsymbol{\tau}, \mathbf{p}) \in \Gamma(\beta)} F(\beta, \boldsymbol{\tau})}, \frac{\max_{(\boldsymbol{\tau}, \mathbf{p}) \in \Gamma(\beta)} G_2(\beta, \mathbf{p})}{\min_{(\boldsymbol{\tau}, \mathbf{p}) \in \Gamma(\beta)} u \left(\frac{1}{p_i} \right)} \right].$$

Because the diameter of measures μ_β goes to zero and $\widehat{\tau}_i \rightarrow 1$ as β goes to 1,

$$\lim_{\beta \rightarrow 1} \max_{(\tau, \mathbf{p}) \in \Gamma(\beta)} G_1(\beta, \mathbf{p}) = \lim_{\beta \rightarrow 1} \max_{(\tau, \mathbf{p}) \in \Gamma(\beta)} G_2(\beta, \mathbf{p}) = 0,$$

so the limit of $d(\beta)$ as $\beta \rightarrow 1$ is zero. Thus, there exists a value β^* such that $d(\beta) < r_U$ for all $\beta \in (\beta^*, 1)$, which completes the proof. ■

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